BARRY HUFKER

Fibonacci Numbers, the "Golden Mean," and Audio Engineering

The famous "golden mean" has influenced mathematicians since the days of Pythagoras. As with Fibonacci numbers, it keeps turning up in the most unlikely places. What about in the world of audio?

IT WAS LEONARDO OF PISA—FIBONACCI, as he is more-popularly known—who gave the world the *Liber Abaci*, or, *Book of the Abacus*, which was instrumental in introducing Arabic numerals to Europe. And, it was a slightly-older Fibonacci (which translates from the Italian as "son of a simpleton") who devised the "Fibonacci Sequence"—a series of numbers which arose from a hypothetical mathematics problem based on the breeding of rabbits. His solution gave the sequence; 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, . . ., n. Each number (except the first) is the sum of the two previous numbers.

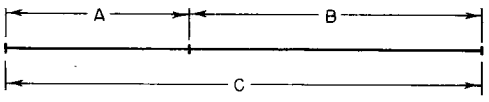
Demonstrating an interesting characteristic (at least to mathematicians), Fibonacci's sequence can be made to yield the ratio, 1.61803 (or, 0.61803), by dividing *any* number in the series, by the number adjacent to it. Although the numbers below 610 only give approximations of these ratios, from 987 onwards, the ratio is precise.

FIBONACCI, AND THE "GOLDEN MEAN"

Long before Fibonacci, this relationship was known as the "golden mean (or ratio)," or, as the "divine proportion." In fact, the work of the ancient Greek sculptor, Phidias, made use of the proportion, and the ratio has come to be known by mathematicians as "ϕ" (phi) in his honor. Perhaps not quite so famous as "π," but. . . .

Phi is the only number whose reciprocal ($1/\phi$) is equal to itself, minus one. In other words, $1/\phi = \phi - 1$. Actually, there are two values of ϕ; 1.61803 and -0.61803. Using the Fibonacci sequence, both values may be found by taking any two adjacent numbers (beginning with 987), and dividing them. Labelling the numbers A and B, we find that A/B = 1.61803, and B/A = 0.61803.

The golden ratio may also be found by dividing a line, C, into "mean," A, and "extreme," B, sections, such that the smaller section is to the greater, as the greater is to the whole. In other words, A/B = B/C, as shown below.



The frequency and diversity with which nature employs the Fibonacci numbers, and the golden ratio, suggests that its universality is more than "just a coincidence." And, Mother Nature has also allowed the Fibonacci proportions to serve both nature and art. Writing in the *Fibonacci*

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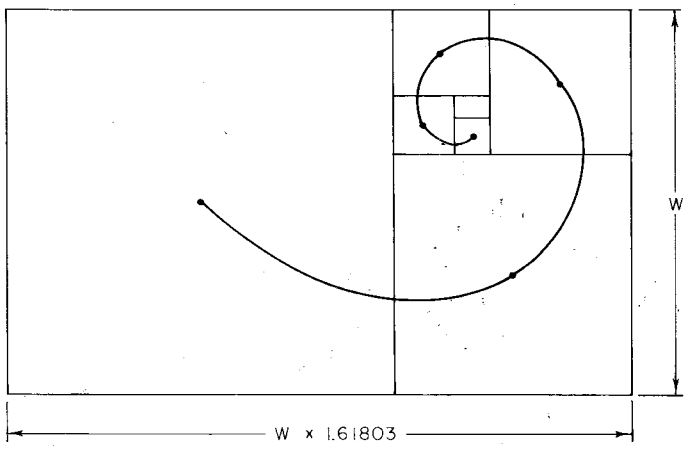
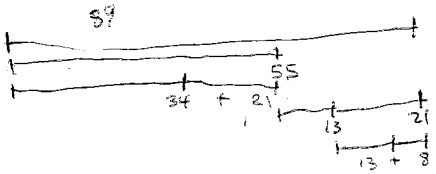


Figure 1. Constructing a "golden spiral." The spiral connects the center-points of progressively-smaller squares. Each square is a section of a golden rectangle.



Quarterly, Marjorie Bicknell and Verner E. Hoggatt, Jr. have reported that, according to German psychologists, ". . . most people do unconsciously favor 'golden proportions' when selecting pictures, cards, mirrors, wrapped parcels and other rectangular objects. In the same publication, Helen Hadian cites artists such as da Vinci, Cezanne, Seurat, Picasso, Gris and many others as users of the golden ratio in their work. Though an extremely detailed discussion would be needed to treat this topic fairly, for the purpose of this article it is sufficient to say that major elements of the paintings (body angle, body proportion and relationships between people and objects, etc.) are based on complex golden mean schemes. Perhaps it has been an intellectual, or instinctive, understanding of geometric proportion which has helped these artists earn their reputations.

THE "GOLDEN SPIRAL" AND THE EAR

An important and interesting outgrowth of the golden mean is the golden spiral, which was investigated by Jakob Bernoulli (who was so impressed, he had it engraved on his tombstone). This logarithmic spiral is derived when one draws a "golden rectangle" (length = width x 1.61803), and then sub-divides it to form a square. The remaining area is a smaller "golden rectangle," and if the process is repeated, a series of progressively-smaller squares are created. A curved line connecting the centers of these squares produces the "golden spiral"—a design that is found again and again in nature. It is the shape of rams' horns, snail shells, the form of galaxies, and as far as audio is concerned, it is the shape of the cochlea of the ear. The entire scientific significance of the cochlea's shape is not completely understood, although it is well-known that people "hear" logarithmically. And, there is strong evidence to demonstrate that, as the Fibonacci proportions appeal to the eye, they also appeal to the ear.

Again in the *Fibonacci Quarterly*, Edward L. Lowman's investigations reveal that ". . . proportion is certainly a major structural and expressive element in music," and, in twentieth-century music, two elements of temporal organization involving Fibonacci numbers stand out. One of these is the "structuring of the lengths of phrases and sections in Fibonacci proportions. The other is the use of Fibonacci numbers . . . to generate what are known as 'irrational' rhythmic values. . . . From the outset, composers found that generating such rhythms from little musical 'games' stimulated their imaginations, assured a measure of consistency, and taught them to free their minds from old and ingrained habits. A chart made from various permutations of the Fibonacci series, a great favorite with many composers, constantly reveals surprising and provocative relationships. In the composer's mind, these are often transformed immediately into musical ideas. . . . Fibonacci proportions have been among the most favored and useful tools."

Use of the Fibonacci sequence and the golden mean is not restricted to any type of music or period of music history. Lowman points out that the composer, Bela Bartók, used them extensively. Often, the golden mean is the major dividing point of a piece. The *Sonata for Two Pianos, and the Divertimento for String Orchestra* are just two such examples. Of the 443 measures of the *sonata's* first movement, Bartók chose the "golden mean" (measure 274) as the place to begin the recapitulation. A similar technique is found in the *Divertimento.*

In Bartók's *Music for Strings, Percussion and Celeste,* Fibonacci numbers are again present, though this time in a more sophisticated manner. The first movement of the piece is 88 measures long, and, according to Lowman, ". . . if we allow a measure's silence, we have 89. The *fff* climax of the movement arrives after 55 measures, of which the strings play the first 34 with mutes, removing them for the last 21. . . . The 34 bars following the climax are divided into 13 and 21 . . . and the final measures are divided again by a change of texture into groups of 13 and 8."

Lowman believes the listener will only subconsciously be aware of the proportions, and yet, "they will do their job just the same. What the listener will perceive is a sense of balance, a feel that the musical events he hears occur at the 'right' places, that they form intriguing patterns in time."

ARCHITECTURAL ACOUSTICS

Within the field of architectural acoustics, the Fibonacci sequence has met with considerable attention, in the design of listening rooms. In Michael Rettinger's book, "Acoustic Design and Noise Control—Volume I," the golden section is cited as one of six ratios that have been frequently used in room design.

In this application, the golden section is given as 1:1.62:2.62. If we take any three adjacent Fibonacci numbers, A, B, C, we will find that B/A = 1.62, and C/A = 2.62 (actually, 1.6803 and 2.61803). Interestingly enough, three of the other five ratio sets are reasonably close to the Fibonacci sequence.

Endless possibilities can be imagined, as various geometric shapes can be realized from the divine proportion. Golden ellipses, cuboids, rectangles, triangles and many more shapes could be selected to vary (and perhaps improved) studio acoustics. It is interesting to note that even the reflection of light rays within two glass plates (a studio window, perhaps) may be expressed in terms of Fibonacci numbers. Can the same be said of sound reflections? At present, the answer is unknown.

FIBONACCI AND BROADCASTING

The author has found the sequence to be of help in producing "King of Instruments"—a weekly program of live organ music and commentary, aired on KWMU-FM, in St. Louis. Golden proportions are utilized in several ways, including the balancing of audio levels.

When recording, the organ is allowed as much dynamic range as possible with the VU meter peaking at approximately 100 per cent (O VU). The voice of the program host is then set to peak at roughly 62 per cent (the golden mean of 100). In doing this a "golden" balance is theoretically established between voice and music which has worked quite well in actual practice.

As a third technique, the divine proportion can be lent to aid the pacing of a radio program, television show or film. In every production there is an attempt made to achieve the proper pace, and get the best balance between various elements. Whether these elements are scenes from a movie or the proper balance between the amount of talk and music in a radio program, there is an attempt to maintain interest for the viewer or listener. The Fibonacci proportions can lend that proper timing. Just as in music, the program can be divided and subdivided into sections whose length are dependent upon their importance and the golden ratios. These sections may be strung together or a single major division can be utilized, in a drama for instance, to realize maximum effect at that point. The possibilities are limitless.

CONCLUSION

It is extremely important to understand that a discussion of this subject is not just a fascination with "Numerology" or some cousin of Astrology. The subjective attraction the golden mean possesses has hopefully been presented as owning some merit. In no way whatsover is it suggested the Fibonacci numbers or the divine proportion be used alone as some sort of "cure-all." No scientifically-proven techniques, research or devices should be discarded. The Fibonacci sequence and the mean are to be viewed simply as tools.

Edward Lowman, in speaking of Bela Bartók (and great composers in general), said that his use of the sequence and the mean as techniques "grew out of the shapes of the musical ideas themselves, just as have most techniques throughout history. One can imagine his realizing at some point that these proportions were what his ideas had been approaching all along. The technique was thus a means of focusing and clarifying the effect." The same can be said of their use in audio.

In music, the foundation for these techniques is already laid. A widespread understanding of such devices makes it hypothetically possible to write "hit" songs based on them or for advertising agencies to come up with more appealing jingles. The theoretical uses in audio of these proportions appear to be vast, though only time and experimentation will determine their true worth. ■